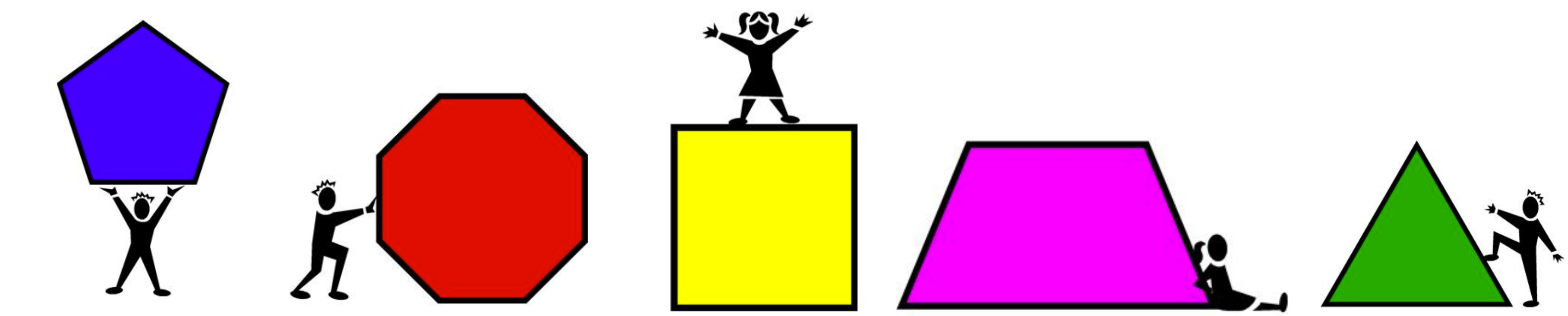
Transformational Geometry

Presenters: Jennifer Hagman & Zachary Hagman



MaThink 2014

March 1, 2014

Who are we?

Zachary Hagman

- I am married to Jennifer.
- I used to live in Colorado.
- I love math.
- I am a pen aficionado.

Jennifer Hagman

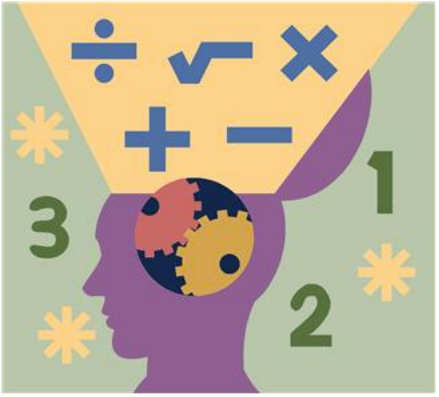
- I am married to Zachary.
- I always wanted to live in Colorado.
- I love math.

We love taking "foot photos".

It's us at MaThink 2014! →

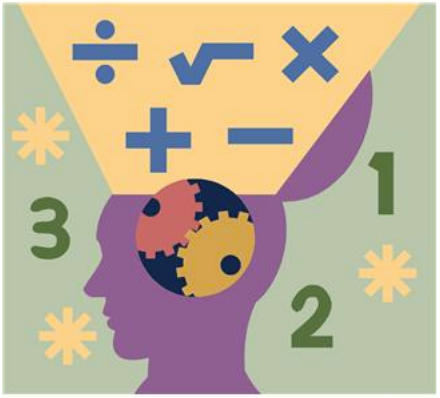
Session Description

Transformational geometry is often viewed statically, but the CCSS showcases it dynamically. This session will explore translations, reflections, and rotations as well as the link between algebra and geometry. No prior geometry knowledge is required.



Brain Drain

- Take a few moments to write down what you know about the following transformations. Feel free to include diagrams, descriptions, mathematical notation, etc., for each term.
  - Translations
  - Reflections
  - Rotations

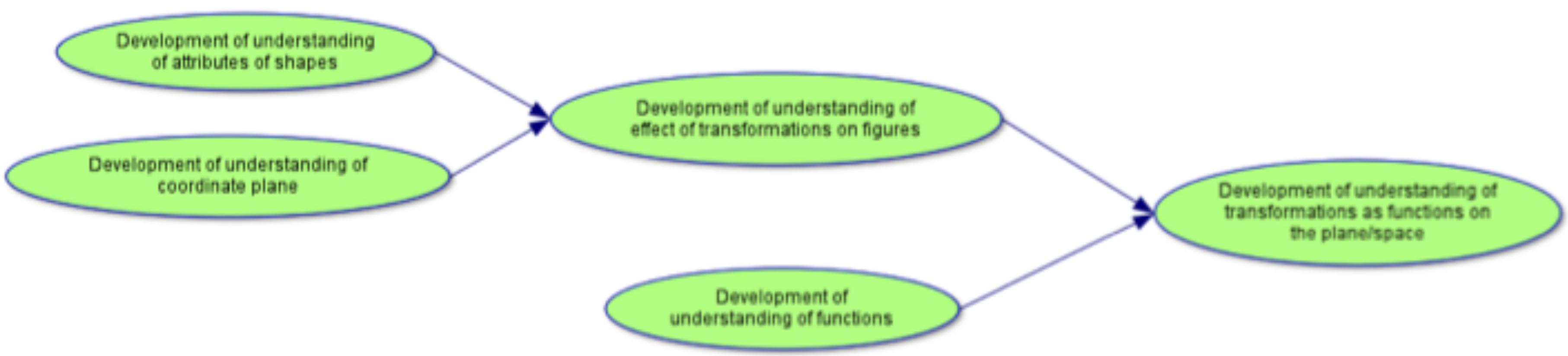


Brain Drain

- Share with the person next to you about the transformation(s) you feel *most* confident about.
  - Translations
  - Reflections
  - Rotations

Transformations

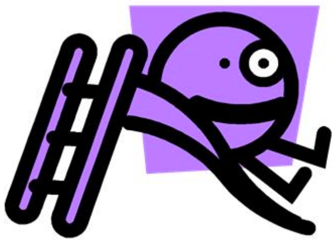
One possible line of development of understanding is shown below.



Source: http://caccssm.cmpso.org/geometry-task-force/geometry-resources

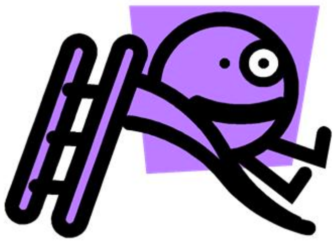
Transformations

- The convention to show a transformation has occurred is to let the original figure be described by a set of points (e.g., A, B, and C) and the transformation of that figure, or image, is described by the mapping of those points to their image points (e.g., A', B', and C').
- For this presentation, we will use this convention, and we will also color code original figures and their images.



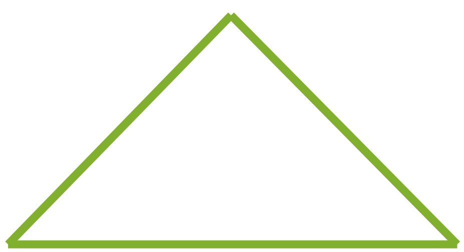
Translations

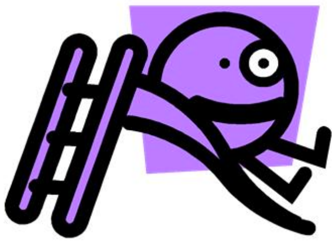
- What are they?
- Translations are often viewed as “slides”.
- The green triangle is our original figure. The orange triangle is its image (transformation).
- How can we verbally describe the net effect on the original triangle?



Translations

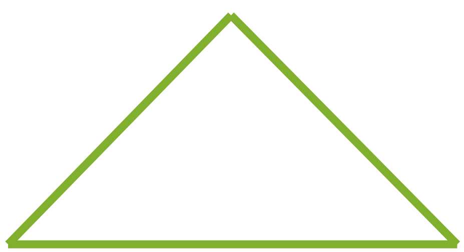
- Consider the triangle below.
- What would it mean to translate it three units?
- How can we perform a translation?

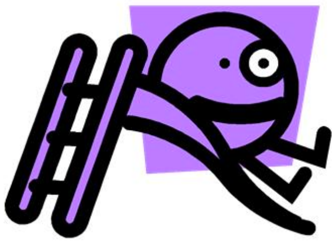




Translations

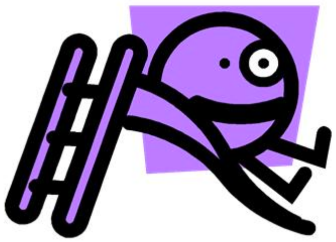
- On the top grid of your paper, draw the following triangle:
  - A is at (1, 1)
  - B is at (2, 4)
  - C is at (4, 1)





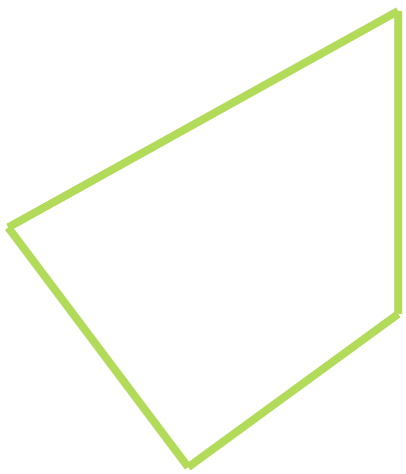
Translations

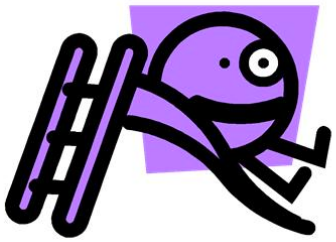
- Given the green triangle ABC, translate it two units right and then six units down.
- Given the same green triangle, translate it six units down and then two units right.
- Are the images in the same location? Why or why not?
- Make a conjecture about the order in which you perform translations.



Translations

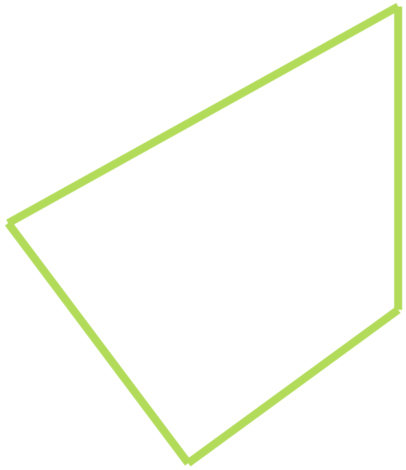
- Consider the following original figure below. Draw it on the second grid on your page.
  - A is at (-6, 2)
  - B is at (-3, 4)
  - C is at (-3, 1)
  - D is at (-5, -1)

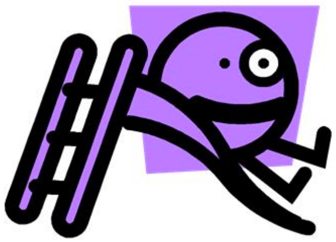




Translations

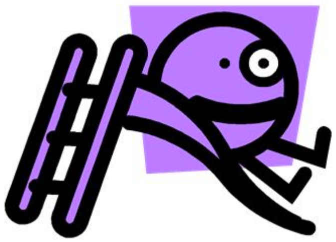
- Using the original figure, perform a translation of four units to the right.
- Compare your image with your neighbor's.





Translations

- What really happened in our last translation?
- How many points "moved" to a new location?



Translations

- What translation would you need to perform so that image of point A of the green triangle is at the origin (0,0)?
- What translation would you need to perform so that the image of point D of the green quadrilateral is at the origin?

Connections to Algebra

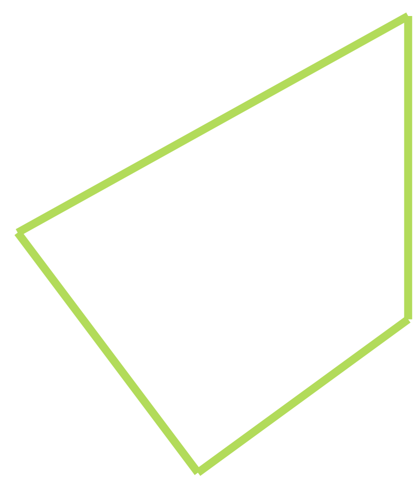
- How can we see translations algebraically?
- Consider the green quadrilateral.
- Compare the points in the original figure to the points in its image.
- $f(x, y) = (x + 4, y)$

Connections to Algebra

- Compare the points in the original green triangle to the points in its image.
- How can we represent our translation of our green triangle algebraically?
- $f(x, y) = (x + 2, y - 6)$

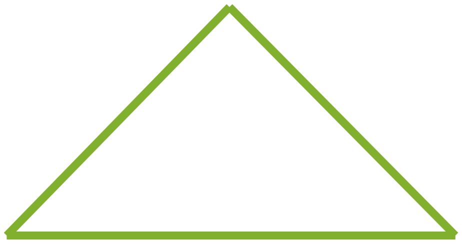
Connections to Algebra

- Consider again our translation of the green quadrilateral so that image of point D is at the origin.
- How could we write this translation algebraically?
- $f(x, y) = (x + 5, y + 1)$



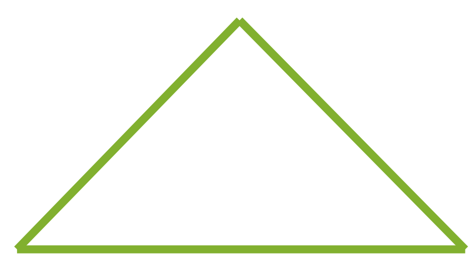
Connections to Algebra

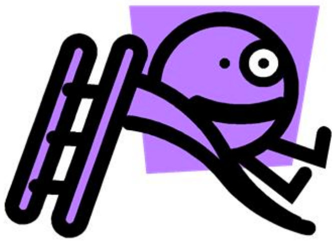
- Consider again our translation of the green triangle so that image of point A is at the origin.
- How could we write this translation algebraically?
- $f(x, y) = (x - 1, y - 1)$



Connections to Algebra

- Consider the following translation of our green triangle: f(x,y) = (x+2, y-6)
- We could also think of translating (sliding) our triangle along a vector parallel to < 2,-6 >.

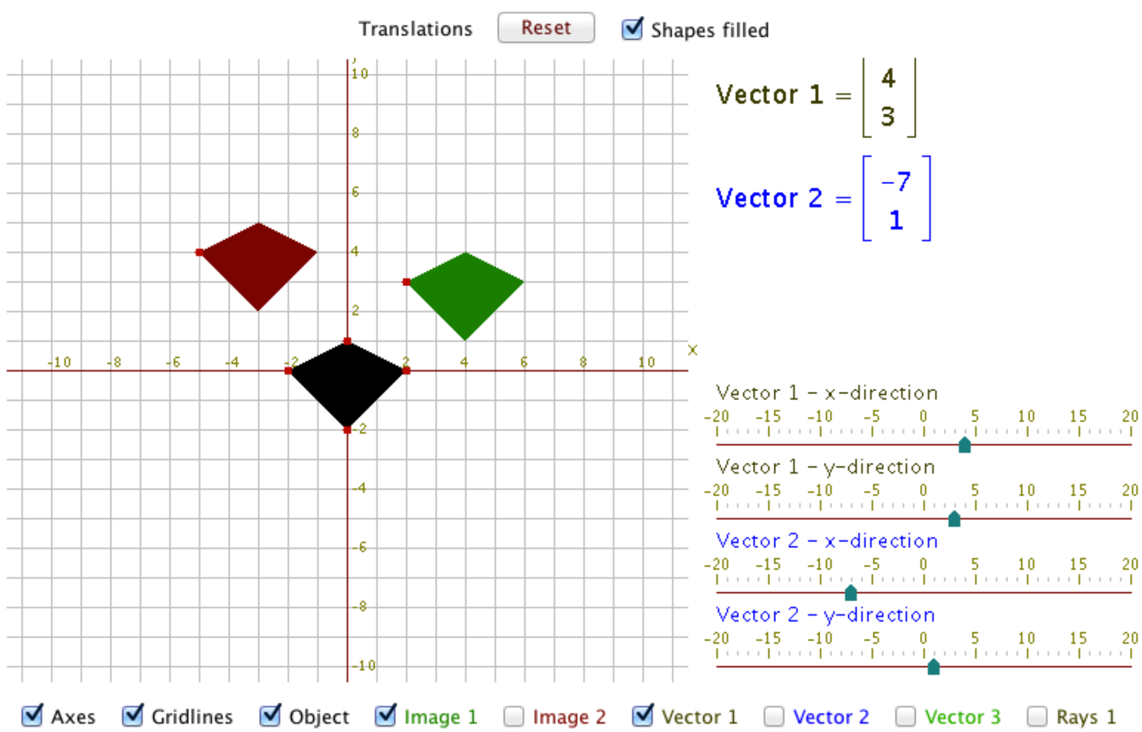


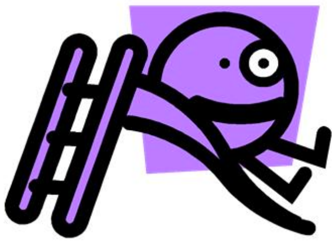


Translations

- The following website allows the user to view translations dynamically.

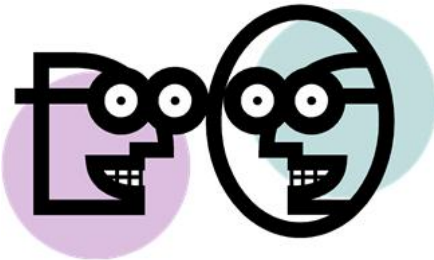
  http://www.waldomaths.com/Translations1L.jsp





Translations

- Take a moment to fill in the box at the bottom of the “Translations” page with an a-ha you had.
- What have you learned about translations?

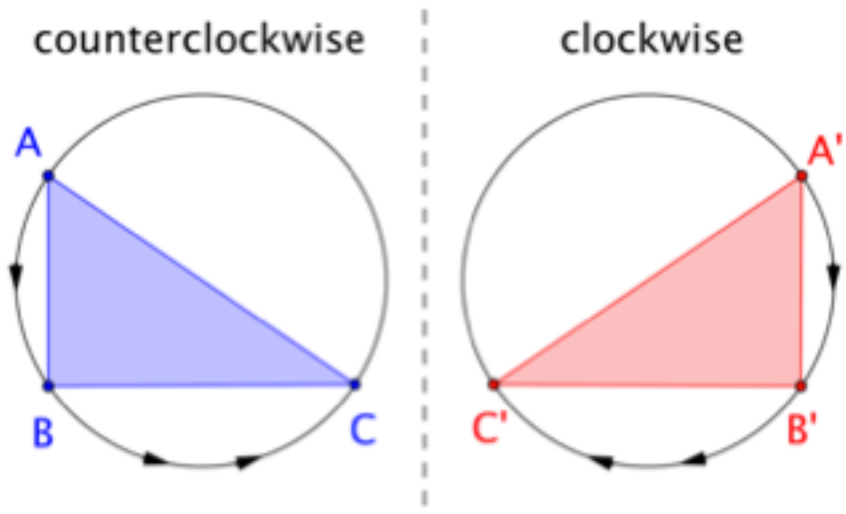


Reflections

- What are they?
- Reflections are often thought of as “flips”.
- Use your patty paper to perform a reflection of the green triangle over the y-axis.
- What is the same or different about the two triangles?

Reflections

- A positive orientation is one that goes counterclockwise. This is a mathematical convention.
- A negative orientation is clockwise.



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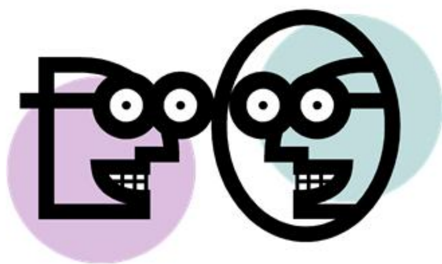
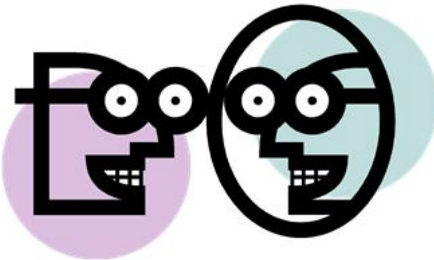


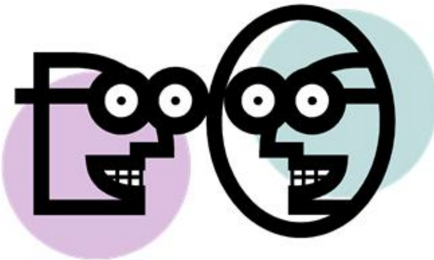
Reflections

- Consider the green triangle that was reflected over the y-axis.
- Take this image triangle and reflect it over the x-axis.
- What observations can you make about this new triangle (A’’B’’C’’)?



Reflections

- What do you think would have happened if you had first reflected the green triangle over the x-axis, and then reflected the image over the y-axis?
- Use your patty paper to test your conjecture.



Reflections

- Consider the green quadrilateral.
  - A is at (-1, 3)
  - B is at (2, 5)
  - C is at (2, 2)
  - D is at (0, 0)
- Reflect it over the y-axis.
- Now reflect it over the x-axis.

Connections to Algebra

- How can we see reflections algebraically?
- Consider the green quadrilateral.
- Compare the points in the original figures to the points in the image.
- $f(x, y) = (-x, -y)$

Connections to Algebra

- Compare the points in the original quadrilateral to the points in its image.
- How can we represent the reflection of our green quadrilateral algebraically?

Reflections

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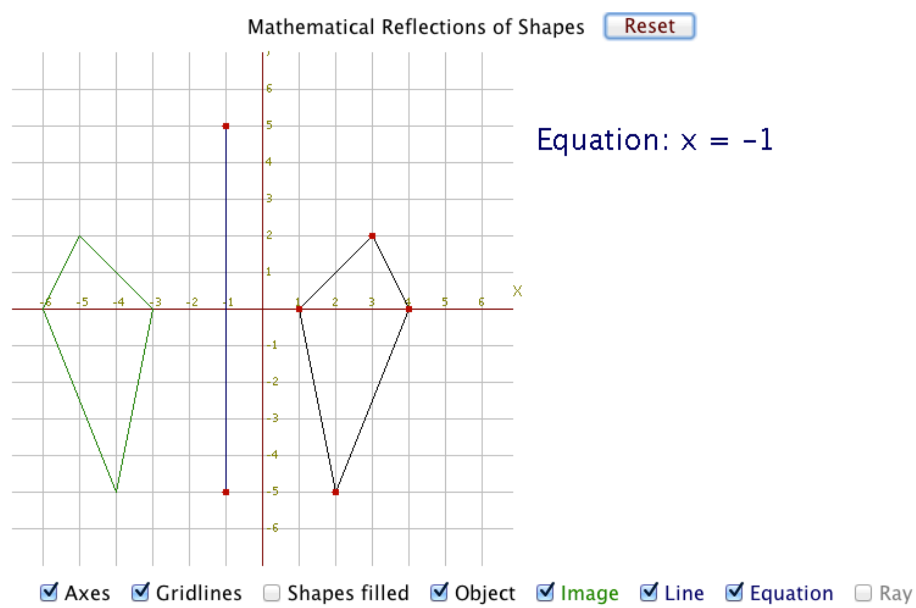

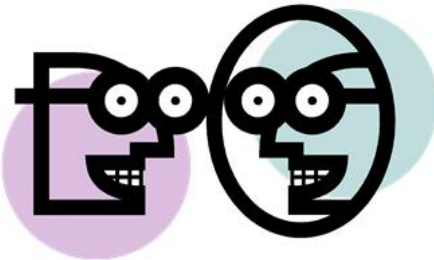

Reflections

- Take a moment to fill in the box at the bottom of the “Reflections” page with an a-ha you had.
- What have you learned about reflections?

Rotations

- What are they?
- Rotations are often thought of as “turns”.
- How can we use our patty paper to perform a rotation?

Rotations

- Perform a 180 degree rotation of the green triangle.
- Perform a 90 degree rotation of the green quadrilateral.
- Perform a 90 degree *positive* rotation of the green quadrilateral.

Connections to Algebra

- How can we see rotations algebraically?
- Consider the green quadrilateral.
- Compare the points in the original figures to the points in the image.

Connections to Algebra

Original Quadrilateral

- A (-1, 3)
- B (2, 5)
- C (2, 2)
- D (0, 0)

Image Quadrilateral

- A' (-3, -1)
- B' (-5, 2)
- C' (-2, 2)
- D' (0, 0)

We could think of this as a function: f (x, y) = (-y, x).

We could also think of this as matrix multiplication:

f (x, y) = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Rotations

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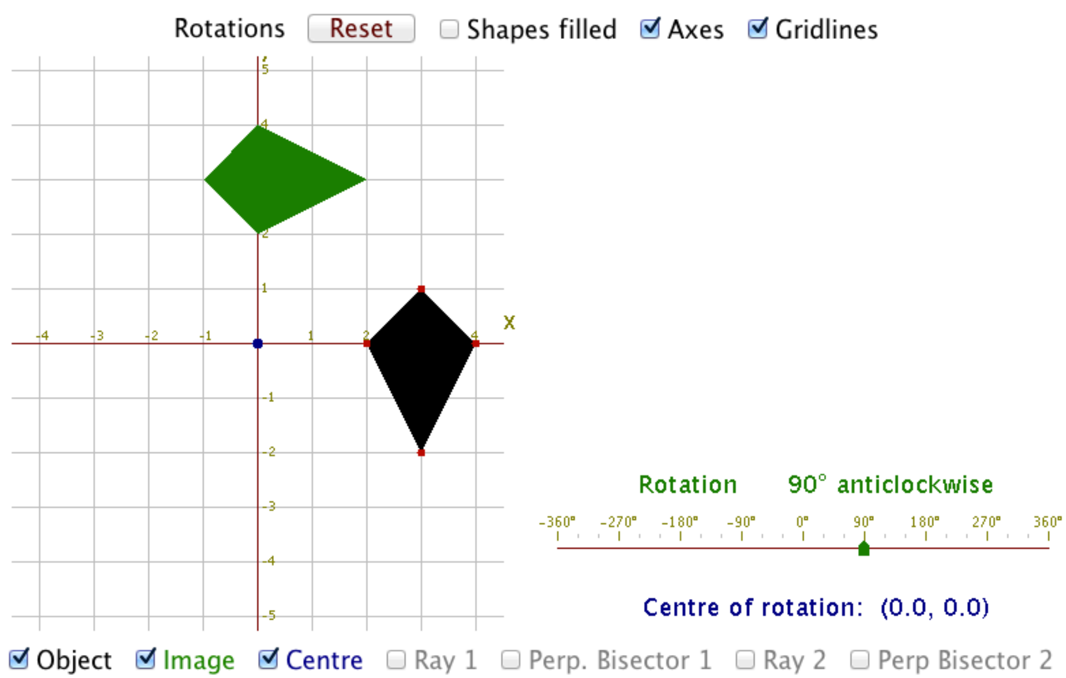
  - http://www.waldomaths.com/Rotations1L.jsp



Rotations

- Take a moment to fill in the box at the bottom of the “Reflections” page with an a-ha you had.
- What have you learned about reflections?

Connections to Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Connections to CCSS Progressions

The Common Core State Standards in mathematics were built on progressions, that is, narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. These documents trace major concepts through the grade levels. They explain which standards build upon one another, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics.

Connections to CCSS Progressions

K-12 Domains

<table>
<tr><th>K</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><td colspan="9">Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="3">The Number System</td></tr>
<tr><td colspan="6">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td></tr>
<tr><td>Counting and Cardinality</td><td></td><td></td><td colspan="3">Number and Operations---Fractions</td><td colspan="2">Ratios and Proportional Relationships</td><td>Functions</td></tr>
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High School Conceptual Categories

NUMBER AND QUANTITY	ALGEBRA	FUNCTIONS	GEOMETRY	STATISTICS AND PROBABILITY

From: http://www.illustrativemathematics.org/

Selected CC Geometry Standards

- Grade 5
  1) … Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axis and the coordinates correspond.
- Grade 7
  1. Solve problems involving scale drawings of geometric figures, including … reproducing a scale drawing at a different scale.

Selected CC Geometry Standards

Grade 8

1. Verify experimentally the properties of rotations, reflections and translations.
   a. Lines are taken to lines, and line segments to line segments of the same length.
   b. Angles are taken to angles of the same measure.
   c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

References

- CMP Geometry Task Force Materials (http://caccssm.cmpso.org/geometry-task-force/geometry-resources)
- Interactive Geometry Websites:
  - Translations: http://www.waldomaths.com/Translations1L.jsp
  - Reflections: http://www.waldomaths.com/Reflections1L.jsp
  - Rotations: http://www.waldomaths.com/Rotations1L.jsp

Thank you for attending!

- This presentation and handouts are available at:
  - http://algebraforum.wikispaces.com/
- Feel free to contact us via email.
  - Jennifer: Jennifer.R.Hagman “at” gmail.com
  - Zachary: Zachary.Hagman “at” gmail.com